

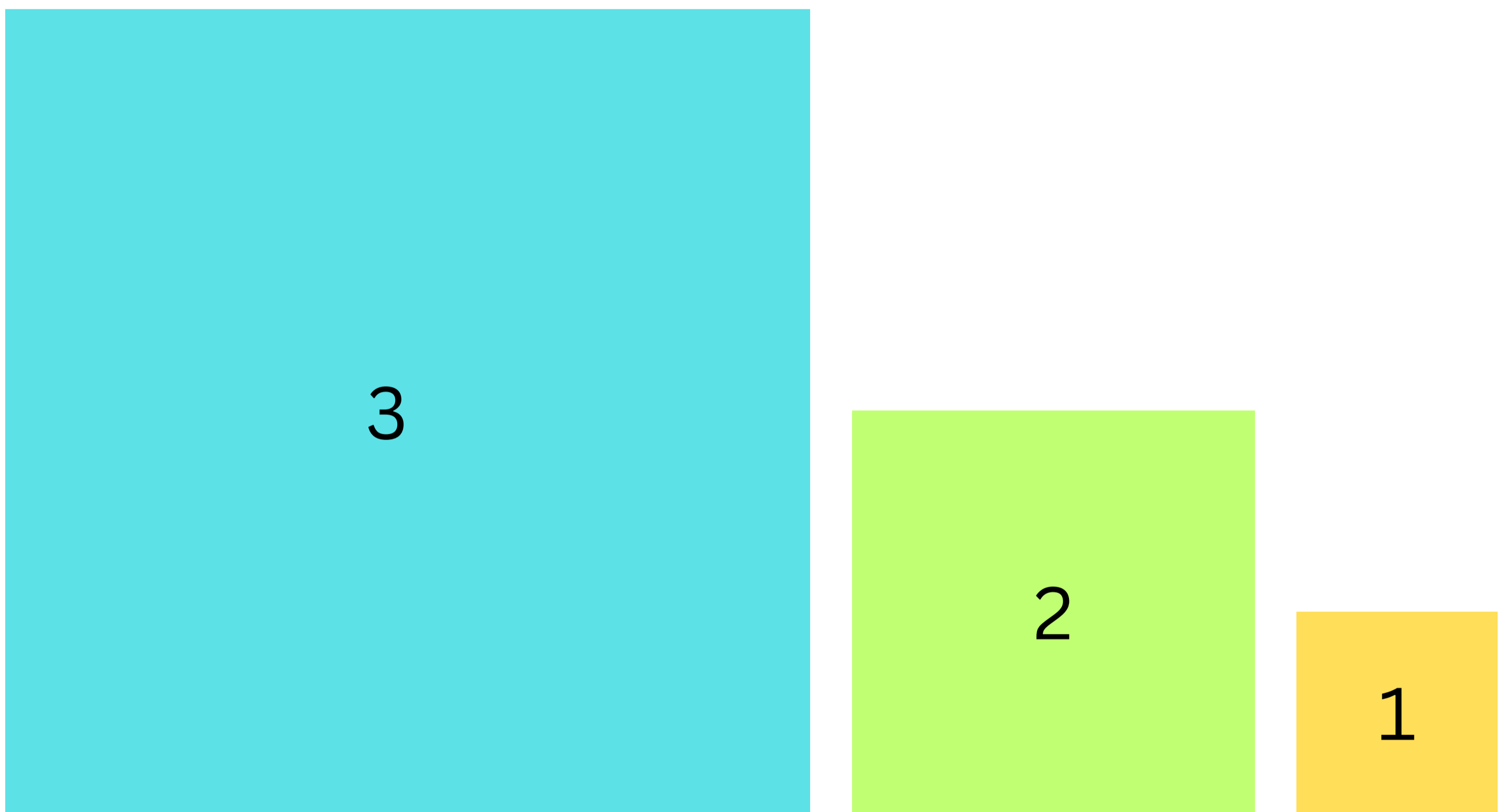
Math Inquiry Task

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Supersized Squares

You will be given 3 pieces of paper, like the ones illustrated below. First working on your own, and then working in groups, figure out how the size of the papers are related to one another.



Try to show your thinking in as many ways as you can! Draw it out, talk it out, act it out, etc.

Be creative and convincing!
Use any materials you'd like.



Extend Your Understanding

Welcome back!

Now you have learned the word “ratio”.

Ratio is...

the quantitative relationship between two amounts showing the number of times one value contains or is contained within the other.

Consider the following questions:

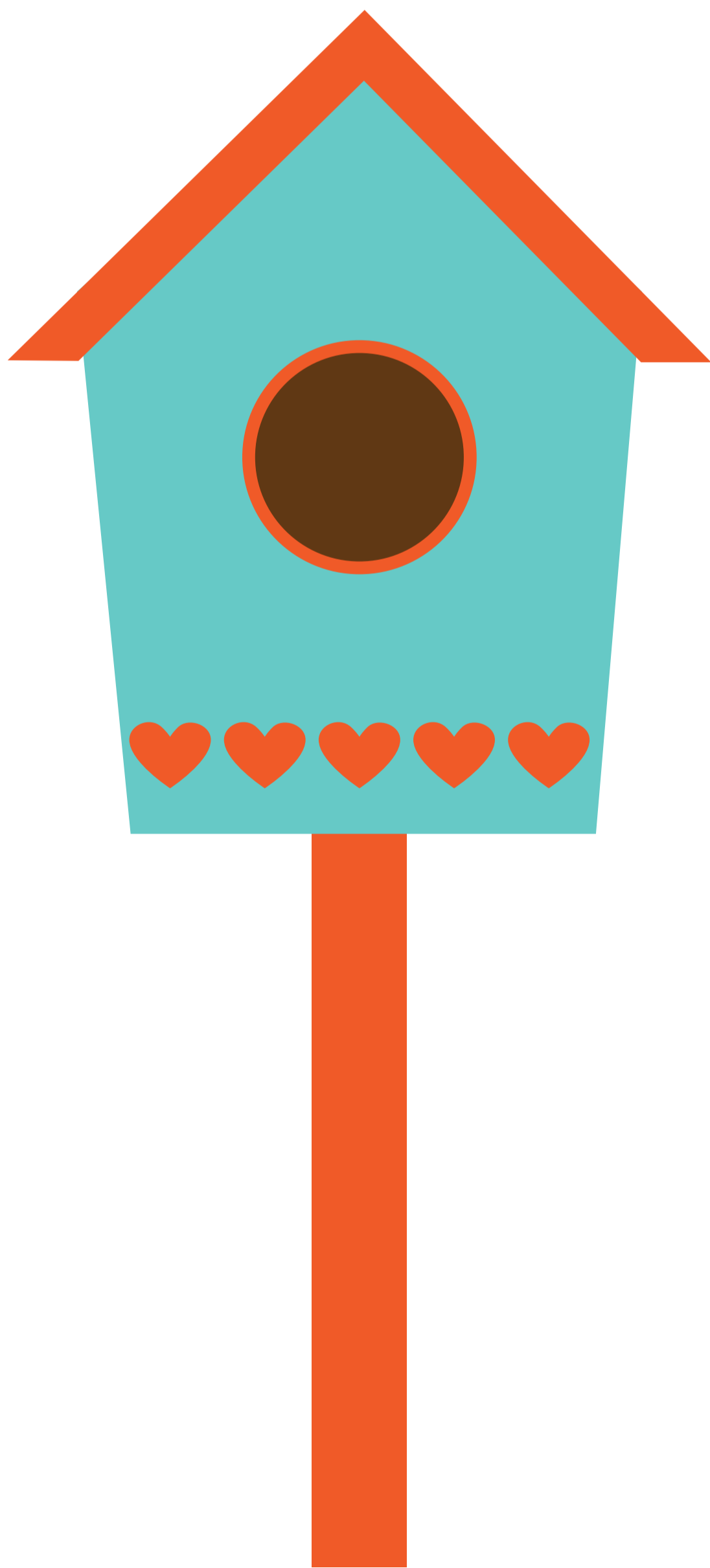
1. What is the ratio between each of the squares?
2. What would be the size of square 0 and square 5 be? Demonstrate your understanding.
3. What would the area of the 5th square be?
4. What is the ratio of side length to area?

Making Paper Birds

You want to make a paper bird for your bird house.
What's the biggest paper square you can use for
your bird?

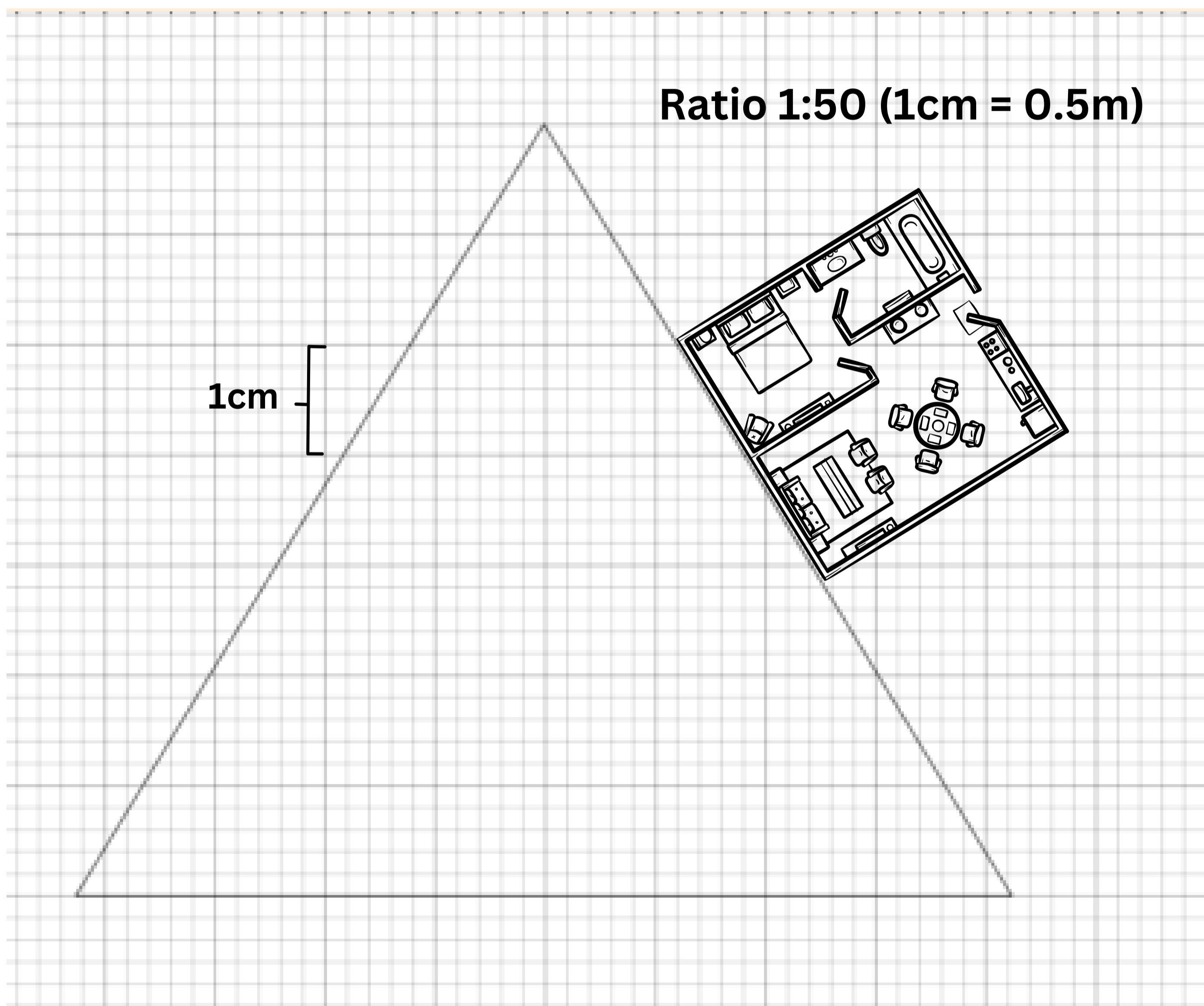
Show your process.

Let's start a conversation! What do you think?
Let's go and measure! What did you find?



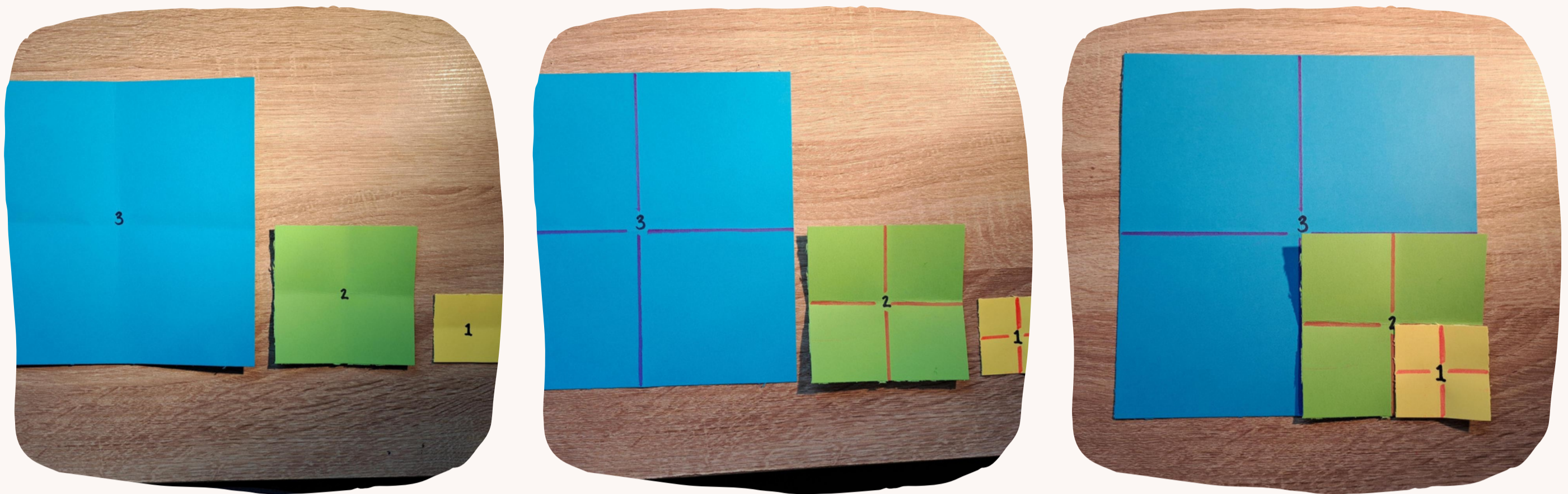
Blueprints

Currently, you're farming within the triangle area outlined below. However, you've recently made a new purchase that allows you to farm 50 square meters total



How would you expand your land?
What would it look like? How much more farming area did you add to make 50 squared meters?
Explain/show your rationale.

Sample Answers



When students are first presented with the squares of paper, they are asked to find how the sizes of the squares are related to one another. This is an open-ended inquiry task because students are not given any measurements or specific tools to use. They can use any methods or tools they want to show the relationship. In this example, the student has chosen to fold the squares into quarters, draw lines on the squares, and then place each square on top of one another.

The student is then able to apply their conceptual understanding to the extension questions. They relate the sizes of the papers to ratios, fractions, and area.

Page 2: Extend your understanding

①

I know that the ratio between square #1 and square #2 is 4:4. The ratio between square #2 and square #3 is also 1:4.

I know this because I folded the squares into fours and found that each square is $\frac{1}{4}$ of the next biggest square.

→ Square #1 is 1:16 square #3 because the yellow square could fit into the blue square 16 times.

②

$A_1 = s^2$
 $A_1 = (2 \text{ in.})^2$
 $A_1 = 4 \text{ in.}^2$

$A_0 = (1 \text{ in.})^2$
 $A_0 = 1 \text{ in.}^2$

I know that square #0 would be four times smaller than square #1 because the ratio is 1:4. Therefore if square #1 is a 2 in. x 2 in. square, then square #0 is a 1 in. x 1 in. square.

② continued...

Square #5 would be 4 times larger than square #3.

Square #	side length ⁽ⁱⁿ⁾	area (in ²)
0	1	1
1	2	4
2	4	16
3	8	64
4	16	256
5	32	1024

Therefore, square #5 would be 1024 in.² or 32 in. x 32 in.

③ The area of the 5th square would be 1024 in.²

$A_5 = s^2$
 $A_5 = (32 \text{ in.})^2$
 $A_5 = 1024 \text{ in.}^2$

I could figure this out by measuring the side lengths of square 3 and then multiplying by 4 to get 32 in.

Sample Answers

④ The side lengths increase by a factor of 2.
The areas increase by a factor of 4.

$A = s^2$
↑
formula

ratio of side length to area for Square 0, 1, and 2.

Making Paper Birds

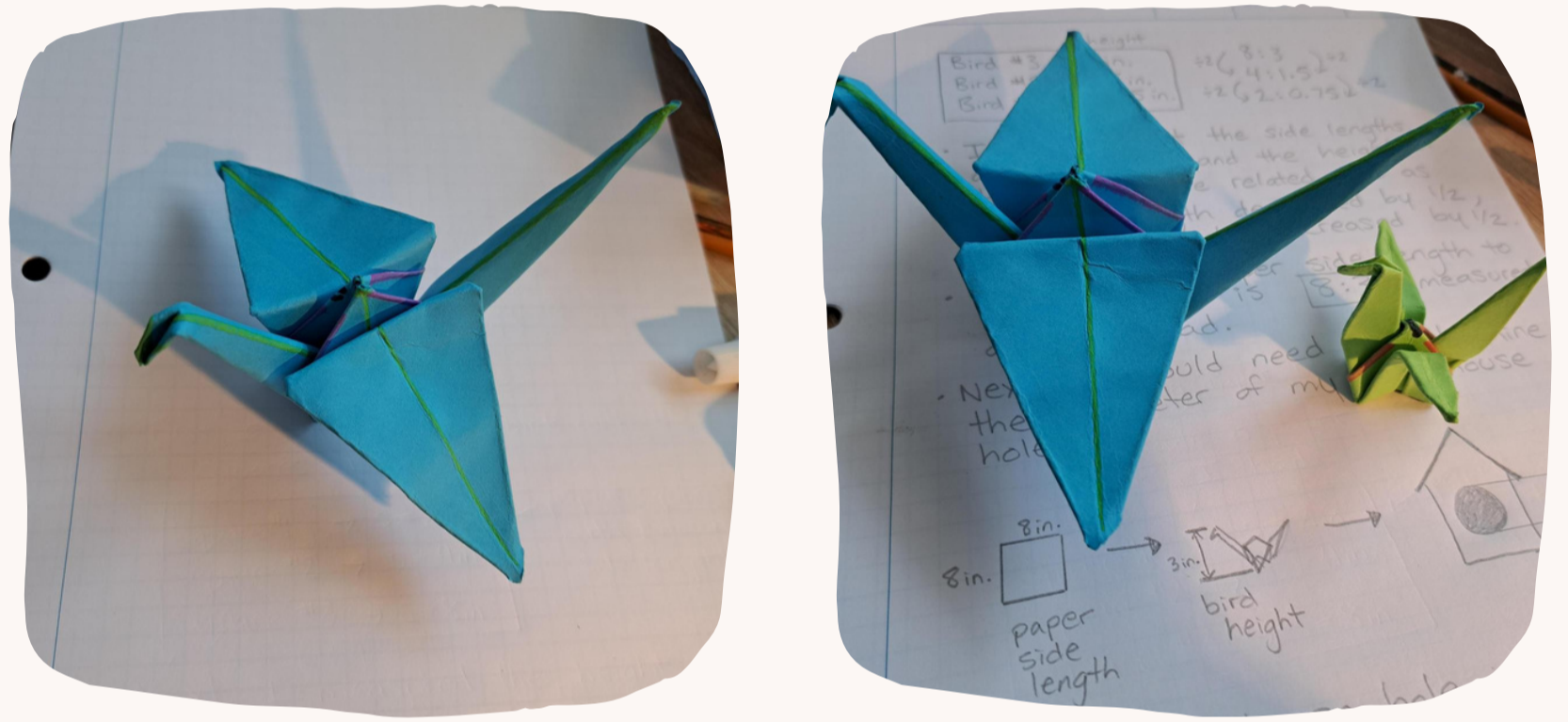
- First, I made 3 paper birds with the 3 squares I had.

Square #1 → $s = 2$ in.
Square #2 → $s = 4$ in.
Square #3 → $s = 8$ in.

← paper squares with known side lengths.

- Next, I measured the height of each bird.

The student is asked to further apply their understanding of ratios to a “real-world” problem of making Origami cranes. Again, the student is given no specific measurements or tools to use. They are given the freedom to demonstrate their understanding in any way they like. This creates a “low floor, high ceiling” dynamic in which students can enter the problem at any level.



This student has chosen to fold paper cranes using the 3 papers they were provided. Using a ruler, they determined the side length of each paper. Then they compared the size of the paper to the size of the corresponding bird.

height

Bird #3 → 3 in. $\div 2 \left(\begin{array}{l} 8:3 \\ 4:1.5 \end{array} \right) \div 2$
Bird #2 → 1.5 in.
Bird #1 → 0.75 in. $\div 2 \left(\begin{array}{l} 2:0.75 \end{array} \right) \div 2$

- I noticed that the side lengths of the papers and the height of the birds were related, and as the side length decreased by $\frac{1}{2}$, the bird height decreased by $\frac{1}{2}$.
- The ratio of paper side length to bird height is $8:3$ measured at the head.
- Next, I would need to determine the diameter of my bird house hole.

8 in. \square → 3 in. \square → \square 3 in.

paper side length bird height

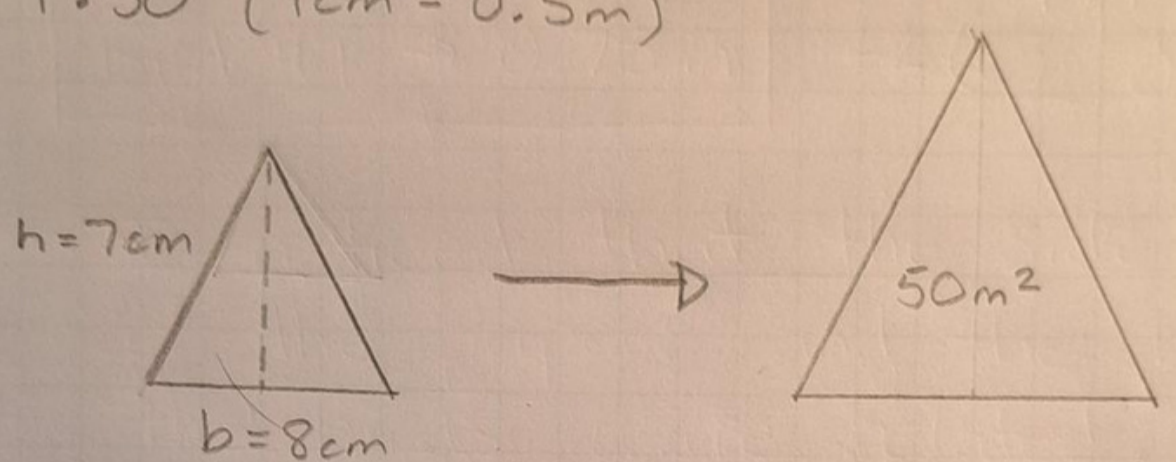
- If the bird house hole is 3 in. \varnothing then, my 8 in. square is perfect.



Sample Answers

Blueprints

1:50 (1cm = 0.5m)



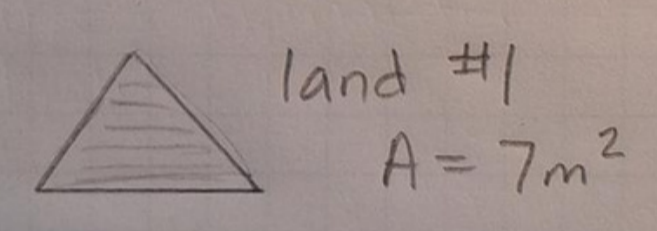
$h = 7\text{cm}$
 $b = 8\text{cm}$

50m^2

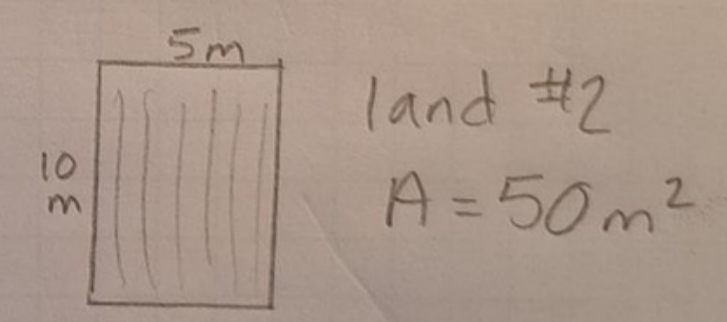
$7\text{cm} \rightarrow 350\text{cm} = 3.5\text{m}$
 $\times 50$

$8\text{cm} \rightarrow 400\text{cm} = 4\text{m}$
 $\times 50$

$A = \frac{bh}{2} = \frac{(3.5\text{m})(4\text{m})}{2} = 7\text{m}^2$

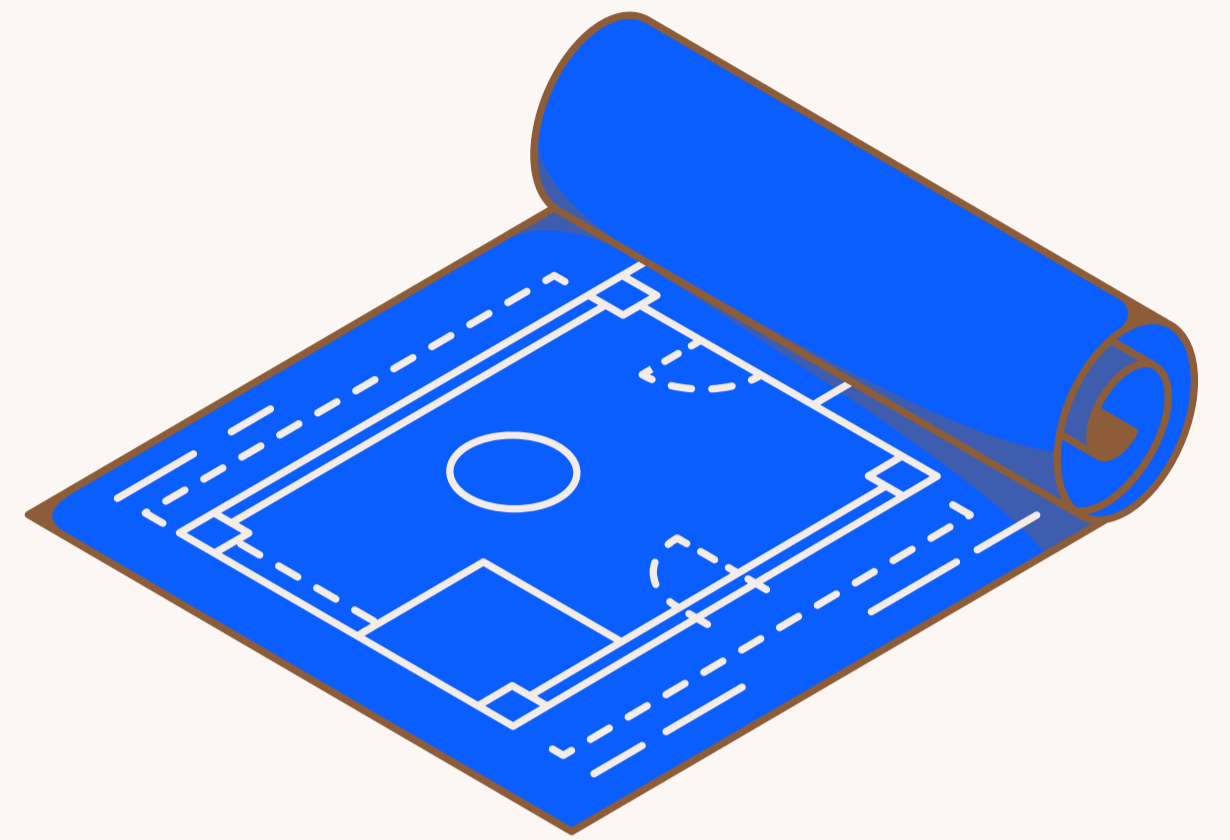


land #1
 $A = 7\text{m}^2$



land #2
 $A = 50\text{m}^2$

Therefore, I have 43m^2 more land to farm now.
I would make it a $5\text{m} \times 10\text{m}$ rectangle because a rectangle is easier to plant in rows than a triangle is. The corners of the triangle would be a waste of space.



Lastly, the student is given another application problem. This one is a common problem found in occupations such as architecture and construction. The student must use ratios to read blueprints, scaling the measurements up.

Curriculum Connections

Grade 6 Math

B2.12. Solve problems involving ratios, including percents and rates, using appropriate tools and strategies.

E2.1. Measure length, area, mass, and capacity using the appropriate metric units, and solve problems that require converting smaller units to larger ones and vice versa.

(See full lesson plan below)

Task(s)

- Supersized Squares
- Extend Your Understanding
- Making Paper Birds
- Blueprints

(See Task Sheets Above)

Mathematic Rationale

Developing engaging mathematics lessons can be challenging, but Jo Boaler (2016) suggests that creating interesting tasks and questions that foster learning is the key to success and engagement in the classroom. Mathematically engaging tasks combine the 5 C's—curiosity, connection making, challenge, creativity, and often collaboration (Boaler, 2016). Our task aims to engage students in inquiry using the 5C's while helping them learn and understand ratios.

The “Supersized Square” task provides an accessible low-floor, high-ceiling starting point that removes the stress associated with mathematics as it invites students to think visually about the evolving shape. This task is “low floor” because it builds on an intuitive understanding

of proportions, that students can conceptualize, without necessarily knowing the related definitions or formulas. Students are initially given 3-5 minutes to work, allowing for individual sketching of their unique process towards a solution.

Through our task, students can cultivate a growth mindset as they work in heterogeneous groups and share different solutions. The cumulation of solutions promotes the amalgamation of different perspectives and concepts, fostering an environment of cooperation and reducing the apprehension of making errors. Our grouping decisions promote engagement and excitement (Boaler, 2016), as different group dynamics develop peer connections, reinforce respect, and promote active listening.

As an inquiry task introducing ratios to sixth-grade students, the objective is to present a challenge while still being accessible. The degree of difficulty is influenced by the student's prior knowledge, age, and grade level (Smith & Stein, 1998). Since it is a problem without a clear-cut method or formula, it introduces the concept while encouraging the students' creativity or connection to existing knowledge. In addition, by initially challenging the students, engagement is shown to increase when learning the method subsequently (Boaler, 2016). Once students complete this task, we invite students to share their discoveries and conclusions to create a student-generated definition of ratios. With the newly generated definition, students are encouraged to consider five general questions in the "Extend Your Understanding" worksheet.

The additional reach tasks—"Making Paper Birds" and "Blueprints"—are intended to build upon newly acquired knowledge. The tasks encourage students to "do mathematics" rather than relying solely on memorizing procedures or formulas (Smith & Stein, 1998). These tasks generate opportunities to establish connections between ratios and area. "Making Paper Birds" explores a 3D scale approach to ratios by using origami paper cranes; while "Blueprints"

simulates a real-life approach through scale drawings. To nurture an inclusive learning environment and foster creativity, the students are provided with the necessary supplies such as paper squares, scissors, coloured pencils, rulers, chart paper, markers, and tape.

Through the framework of Jo Boaler (2016) we have aligned our mathematical tasks to incorporate the 5C's. We spark curiosity by intentionally withholding information and scaffolding the various tasks. Our approach to connection-making involves leveraging both abstract and real-life concepts. Open-ended questions stimulate creative thinking, where students explore multiple pathways towards a solution. This methodology challenges students to "do mathematics" by encouraging them to think critically and derive solutions on their own, and in collaboration with others. Overall, our approach is to promote engaging mathematical exploration, which aids students in comprehending ratios.

References

- Boaler, J. (2016). *Mathematical Mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching*. San Francisco, CA: Jossey-Bass. Chapter 2 & 5.
- Smith, M. S., & Stein, M. K. (1998). Reflections on practice: Selecting and creating mathematical tasks: From research to practice. *Mathematics teaching in the middle school*, 3(5), 344-350

Inquiry Task: Ratios

Grade: 6	Time Frame: 2 days	Instructors: Sam Park, Sheena Brennan, Vanessa Li
Subject: Mathematics		
Overall Expectation: B: Number Sense; E2 Measurement		
<p>B2: Use knowledge of numbers and operations to solve mathematical problems encountered in everyday life</p> <p>E2: compare, estimate, and determine measurements in various contexts</p>		
Specific Expectations: B2.12: Operations; E2.1: The Metric System		
<p>B2.12: Solve problems involving ratios, including percents and rates, using appropriate tools and strategies</p> <p>E2.1: Measure length, area, <u>mass</u>, and <u>capacity</u> using the appropriate <u>metric units</u>, and solve problems that require converting smaller units to larger ones and vice versa</p>		
Cross Curricular Expectations: Language Arts		
<p>D1: Developing Ideas and Organizing Content: plan, develop ideas, gather information, and organize content for creating texts of various forms, including digital and media texts, on a variety of topics</p> <p>D1.4 Classify and sequence ideas and collected information, using appropriate strategies and tools, and identify and organize relevant content.</p>		
Learning Goals: Students will...		Success Criteria: Students can...
<ul style="list-style-type: none"> Use an inquiry task to determine the ratio variations among 3 pre-determined sized squares. Learn the term “ratio” and create a definition based on shared student ideas. Work with the concept of “ratio” to answer a variety of questions <p>Reach Tasks 1</p> <ul style="list-style-type: none"> Extend learning beyond ratio on a 2D image to considering ratio on a 3D scale through an inquiry task using paper cranes Organize information generated by the group and share ideas with the class in their desired format <p>Reach Task 2</p> <ul style="list-style-type: none"> Apply their knowledge of ratios to a real-life situation of a blueprint drawing with a scale of 1:50. 		<ul style="list-style-type: none"> Define and understand the concept of ratio. Connect ratios to surface area Use their knowledge of ratios to create an appropriately sized 3D image Work collaboratively with team members Present their findings and clearly communicate and defend their decisions Connect ratios to real-life situations such as blueprints Use ratio to correctly size up or down a blueprint drawing

<ul style="list-style-type: none"> Find the area of a triangle and determine the actual area based on the ratio provided 	
Materials:	Scissors, paper, coloured pencils, chart paper, rulers, coloured papers, worksheets. math journals, pencils, bird house (for reach task), Origami crane video link (for reach task): https://www.youtube.com/watch?v=KfnyopxdJXQ
<p>Prior to starting the activity, explain the following: To complete this activity, you will first work individually and write your findings in your math journal; you will have 3 minutes to complete this. Next you will connect with your group members and each person will discuss their thinking. Once everyone has shared, you will work together to determine a way to communicate your findings to the class.</p> <p>Materials that you have access to are at the front.</p> <p>Write your name and the role you played in the group on whatever format you decide to use to explain your reasoning.</p> <p>You have 10 minutes total to complete the activity.</p>	
Inquiry Task: Small group	Task: Low-floor High-ceiling
Start: Low-floor, High-ceiling	Find the relationship among the 3 squares on the paper
Work individually	Make notes in your journal about your observations
Share your thoughts	Each person in the group will share their connections among the shapes
Collaborate	As a group, students will collaborate and present their findings using any format they want.
<p>During this individual and group time, the teacher can walk around and ask probing questions, or ask for clarification, but they cannot give direct answers. The teacher can answer questions with other questions, in a manner that promotes inquiry and creative thinking.</p>	
Large Group Lesson	Task: Learning about Ratios
Students share their findings. Use students' understanding to transition to teaching about ratios.	How could you express your findings concisely? Help students make connections from their understanding to the concept of ratios Ratios are written as 1:4 in these equations
Creating a definition of a ratio.	Use the students' information to create a student-generated definition of ratios. Write this definition somewhere on the board.
Inquiry Task continued: Small group	Task: Applying knowledge
<p>Explain that this is a challenge and students will use the information that they just learned. Approach this the same way as the previous inquiry task, but now use ratios.</p>	
Work as a group. All students should share ideas and collaborate to find the answers. There can be more	Using all your materials, answer the questions on page 2: Extend your Understanding.

than one way to solve this. This applies the knowledge just gained about ratios.	Students can use any supplies to complete this task.
Work Individually	Look at the questions and answer them in your personal journal without the help of your team. Organize your thinking.
Share your thoughts	In groups, each student will share their thinking, and explain how they arrived at their answers.
Collaborate	Students will collaborate to answer the questions and demonstrate their understanding, in whatever way they want (Visual, discussion, etc).
Share information	Since this task can be completed at different rates per group, students can defend their decisions to the teacher when they are finished.
Reach Task 1: Origami Birds	Reason
Once students have demonstrated their understanding of ratios with the 2D squares, they can attempt the first reach task where they will follow the same steps as before to create a collaborative answer. Note: Students may want to learn how to make an Origami crane before completing this reach task. A video link is provided here: https://www.youtube.com/watch?v=KfnyopxdJXQ	
Origami birds	Apply the knowledge gained from the inquiry task and apply it to a 3D object to solve the Origami Bird Challenge.
Reach Task 2: Blueprints	Reason
Once students have demonstrated their understanding of ratios applied to a 3D object, they can attempt the final reach task where they will follow the same steps to create a collaborative answer: <ol style="list-style-type: none"> 1. Work individually using math journals 2. Share thinking with group 3. Collaborate with the group to create a final answer 4. Demonstrate understanding in whatever way works for the group. 	
Blueprints	Apply knowledge of ratios to a real-world problem of blueprints.
Differentiated Instruction	
Since this task has a low-floor, high ceiling entrance into the activity, students at any level can work with the group to share their thinking and create a more homogenous thinking process.	
Consolidation: 3 minutes	Next week:
Students can share their answers with the class and defend their decisions by answering questions, posed by the instructor or by other students.	Rates. Since ratios compare two quantities of the same object, a Rate compares 2 quantities of different objects (12km/hour)
KWL: Student reflections sheets will help the teacher understand where there may be learning gaps since group work is sometimes less collaborative than appropriate.	