Reading Journey Map

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1 Skemp (2006)

How can you differentiate between relational and instrumental understanding? Why does it matter to differentiate them?

Recall: Mental map analogy

RELATIONAL UNDERSTANDING: KNOWING WHAT TO DO AND WHY.

Relational = understanding math relationships
Focus: Establishing connections, building understanding over time, applying concepts to other problems, gradual increases in complexity, math theory and formula derivations.
Advantages:
- More adaptable to new tasks.
- Easier to remember long-term.
- More intrinsically rewarding

INSTRUMENTAL UNDERSTANDING: RULES WITHOUT REASONS.

Instrumental = using math instruments and instructions
Examples: "borrowing" in subtraction, "turn it upside-down and multiply" for division by a fraction, "take it over to the other side and change the sign" for functions.
Focus: Rote learning, memory, rules, and correct answers
Advantages:
- Usually easier to 'understand'/ faster to learn.
- Can help give students simple "tricks".
- Can boost student self-confidence.

TO-DO

Teach relational understanding first.
Show more than one strategy.
Encourage mathematical thinking.
Encourage students to take their time.

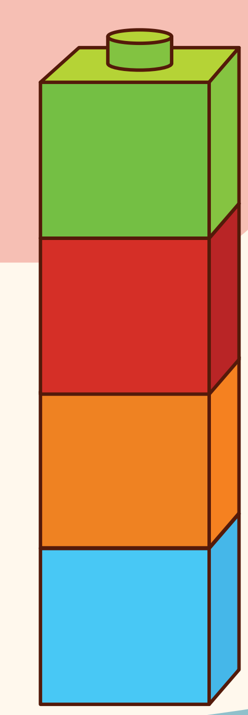
Model math in different ways (draw it, use manipulatives, compare to real life contexts, use numbers and symbols, etc.)!

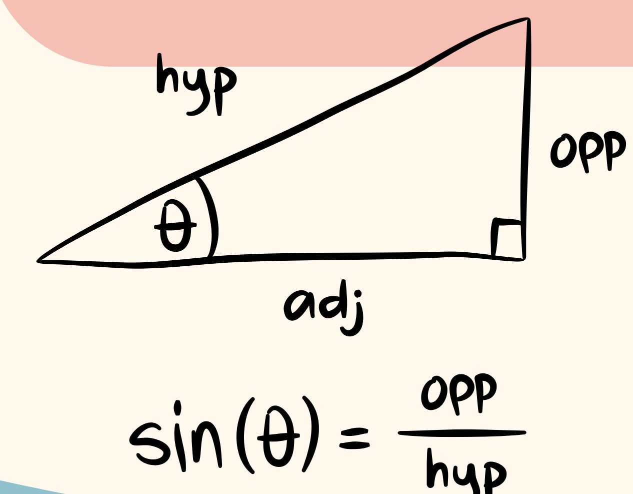
Strand	Meaning	Activities	Rationale
Conceptual understanding	Comprehension of math concepts, relationships, and ideas.	Draw a picture or build a model of the problem. Make connections to other problems.	Students will make less errors when they understand the concepts. Students will remember (retain) the concepts longer than they will remember the procedures.
Procedural fluency	Accurately and appropriately applying mathematical steps (procedures) to solve a problem.	Solve problems in systematic, numerical steps. Follow algorithms.	Algorithms can be the most efficient (fast) way to solve a problem.
Strategic competence	Ability to create, represent, and solve math problems.	Think of a real-world application for a math concept and explain how the problem would be solved. "Math happenings" activity. Book: Jon Scieszka's Math Curse (1995)	Helps students engage with and connect to math problems.
Adaptive reasoning	Being able to explain and justify a solution to a problem.	"Convince me" and "Poster proof" activities.	Allows students to defend their answers, if their answers are questioned. Helps others understand their thinking.
Productive disposition	Tendency to view math positively. Seeing math as useful and worthwhile, and having confidence in one's own mathematical abilities.	Shown by starting on, and preserving through, difficult math problems. Heard by speaking positively about math and explaining ideas to others.	Students will be more engaged with, and excited by math, when they have a productive disposition about it.

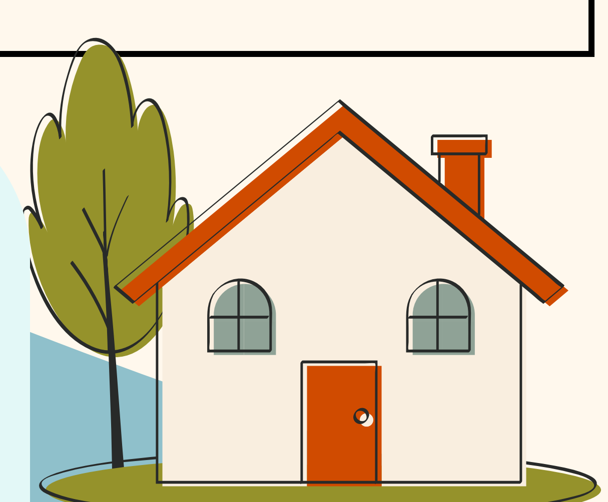
2 Groth (2017) and Suh (2007)

What is the relevance of each strand of math proficiency? What are some strategies/activities to observe and foster math proficiency? Why/how do these strategies/activities develop students' math proficiency?

5 strands of mathematical proficiency
1. Conceptual understanding
2. Strategic competence
3. Procedural fluency
4. Adaptive reasoning
5. Productive disposition







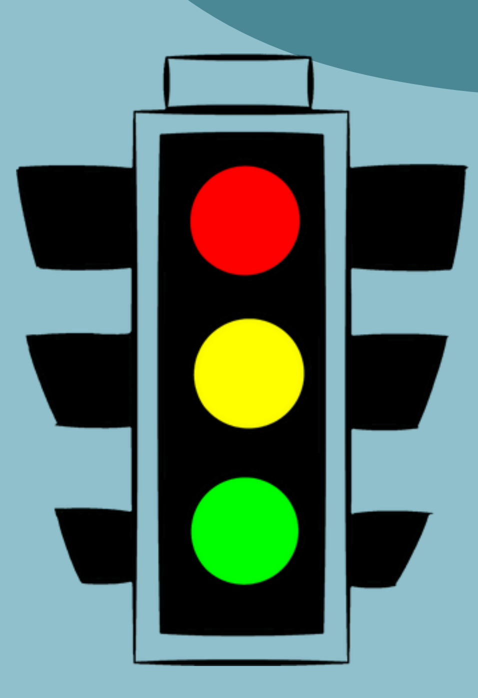
3 Boaler (2016) Chapter 8

Why should teachers consider varied forms of assessment in math? What are some possibilities to challenge traditional math assessments? What is the role of comments in assessments?

WHY VARIED ASSESSMENT:
- More independent learning and self-reflection.
- Students develop intrinsic motivation to achieve academically.
- Increased self-efficacy and growth mindset.
- Accommodate different learning styles.

Assessment for learning: Inform students' learning with feedback
Assessment as learning: Student self-assessment
Formative: Assessment during the learning process, for the sake of learning.

HOW TO ASSESS:
- Assessments without grades, only comments.
- Assessments FOR learning.
- Self-assessment and peer-assessment.
  - o Faces, or thumbs-up.
  - o 2 Stars and a Wish.
  - o Reflection-based exit ticket.
  - o Traffic light assessment.
  - o Drawing a picture to show understanding.
  - o Student written questions and tests.



ROLE OF COMMENTS:
- Provides helpful feedback.
- Students learn faster when given comments and feedback.
- Informs students where they are at and how to improve.
- Positive feedback, with growth mindset messages, increases intrinsic motivation to learn.

4

Boaler (2016) Chapter 7

What are the benefits of teaching heterogeneous groups? What task and instructional strategies can be used when teaching heterogeneous groups? How is each of these strategies relevant?

BENEFITS OF MIXED-LEVEL GROUPS:

- All students are given the opportunity to learn (OTL).
- Encourages growth mindsets.
- Increases student engagement.
- Leads to higher levels of academic achievement.
- Creates more diverse social dynamics and perspectives.

INSTRUCTIONAL STRATEGIES:

- Pose **open-ended, low floor, high ceiling questions** that are accessible to all students, interesting, and engaging.
- Allow students to **work at their own pace.**
- Have **challenge or extension questions** as an option for all students. Students should be able to choose the level of difficulty.
- Ask students to represent their ideas in multiple ways (e.g., words, graphs, symbols, diagrams, pictures, etc.).
- **Math as multi-dimensional:** Encourage questioning, idea generation, making connections, different forms of representing math, and reasoning.
- **Group roles:** Each student should have a job or role within the group. All students should have a purpose, contribute to, and be dependable for learning within the group. Teachers may need to teach students how to work respectfully within a group.
- **Assigning competence:** When appropriate, praise students in front of their peers to validate them and increase their intellectual "status".

WHY DIFFERENTIATE THEM?

As teachers, it is important to know which information is arbitrary, and which is necessary because arbitrary information must be given or assumed, and necessary knowledge must be worked out. If necessary knowledge is given, then it becomes "received wisdom" to be memorized – essentially becoming arbitrary. If arbitrary knowledge is withheld, then students may forget the information, need to make something up, or feel stuck and unable to proceed with the question. Letting students solve for necessary information is more cognitively demanding, memorable, and engaging.

Arbitrary Knowledge	Necessary Knowledge
Names, definitions, or anything else socially/culturally agreed upon, is arbitrary.	Properties, relationships, and anything else that can be determined, is necessary.
Arbitrary knowledge requires memorization.	Necessary knowledge can be figured out with logic and reasoning.
Teachers need to tell students information that is arbitrary.	Students should have the opportunity to figure out information that is necessary.

5

Hewitt (1999)

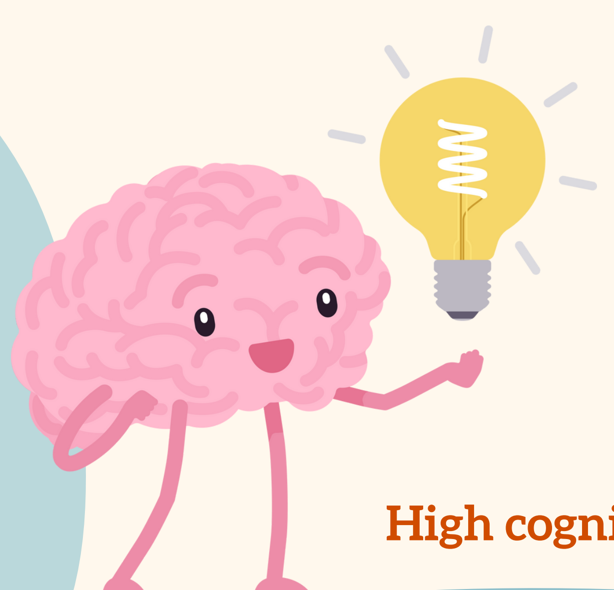
What is the difference between arbitrary and necessary knowledge? Why does it matter to differentiate them?

Doing mathematics: Challenging math tasks, with no clear answer or pathway to get the answer, leads to greater cognitive demand and engagement.

6

Smith & Stein (1998)

How do math tasks relate to cognitive demands and engagement? How is the article's proposed categorization relevant to teachers' practice? What should teachers aim for in math classes? Why?



The proposed categorization of math tasks, from low to high cognitive demand, was: Memorization, procedures without connections to concepts, procedures with connections to concepts, and doing mathematics. This is relevant to a teacher's practice because they can change the question to make it more, or less, cognitively demanding for students. Ideally, teachers would start with high level, complex tasks. It is also important to consider the prior knowledge of the students.

High cognitive demand is not a guarantee for engagement, but it is a necessary first step.

7

Boaler (2016) Chapter 5

What are some strategies when creating/selecting rich mathematics tasks? Why/how is each of these strategies relevant?

- **Open the task.** Allow students to show their understanding in multiple ways.
- **Make the task inquiry based.** Ask questions that require students to think about math concepts and ideas. Allow students to ask their own questions and become curious about math concepts and connections.
- **Ask questions before giving answers.** Give students problems to solve before teaching them the information that would help them solve it. Teach the information they need, as they need it.
- **Use visuals.** It is always helpful to have a visual component, either in the question or in the solution, or both. Use diagrams, pictures, graphs, manipulatives, and more. Encourage students to use colour-coding in their answers whenever possible, for conceptual clarity.
- **Make questions low floor, high ceiling.** To lower the floor, ask open-ended questions like, "What do you think about this problem? What do you notice?". To raise the ceiling, teachers could ask students to write similar, but more challenging questions.
- **Have students convince a skeptic.** Ask students to explain their thinking, in a way that would convince even the most skeptical of people. Ask them to convince themselves, convince their friend, or convince a mathematician.

The 5 C's of mathematical engagement: curiosity, connection making, challenge, creativity, and collaboration

Tasks

8

Boaler (2016) Chapter 2

What are the benefits of teaching through mistakes? What are some strategies to teach through mistakes?

How to know if it's a mistake (careless, procedural error) or a misconception (lack of understanding)?
- **Look at previous work**
- **Ask the student Phrase questions as "explain your reasoning", so that there is more opportunity to see a misconception vs. mistake.**

BENEFITS OF MISTAKES:

- Mistakes mean we are still learning.
- Mistakes activate ERN and Pe responses in the brain.
- Being comfortable with making mistakes, and having a growth mindset, increases the chances of success in life.

STRATEGIES:

- Growth mindset activities.
- Have students share their mistakes.
- The teacher can share common mistakes.
- Model mistakes as the teacher.
- Ask students to create a mistake on purpose.
- Make finding mistakes in a piece of work a regular practice, like a puzzle or a challenge.
- Change the terminology around mistakes. Teachers can use phrases like, "that's a beautiful mistake", and, "your brain must have grown a lot from that mistake."
- Abandon traditional testing and grading.
- Make work challenging enough that conceptual mistakes will be common.

9 Holm (2018)

What are some lesson and unit planning strategies? Why/How is each of these strategies relevant? What is the importance of learning goals?

Unit Planning Strategies

- When starting a unit plan, first identify the following: Curriculum expectations, learning goals, anticipated student struggles, and any prior knowledge students should have.
- Planning should begin by looking at the end goals first. Know what the curriculum says students should learn, and then turn the specific curriculum outcomes into learning goals.

Learning Goals

A learning goal is...

- Based on the curriculum expectations.
- Scaffolded and builds on previous learning.
- Written in simple language that students can understand.
- Actionable and observable. Learning goals should include verbs that can be assessed (e.g., identify, solve, convert).
- Written from the perspective of the student (e.g., I will).

Learning goals are important because they help students take control of their own learning. When students know what they should be learning, then they can self-assess if they have achieved those goals.

10 Boaler (2016) Chapter 6

Why should teachers care about equity in math education? What are some strategies when teaching for equity in math classes? How is each of these strategies relevant?

WHY SHOULD TEACHERS CARE ABOUT EQUITY IN MATH?

All students have the right to education, regardless of their gender, race, socioeconomic status, or any other characteristic. Yet, math is the most unequitable subject taught in the United States, with huge differences in achievement based on socioeconomic status, gender, race, etc. Women, Latino, and African American students are highly underrepresented in math. The discrimination is not obvious but is rather an accumulation of unfair practises. Underrepresented students could love and succeed in math, but they are not given the opportunities they need.

STRATEGIES:

- **Offer all students high-level math content.**
  - De-streaming.
  - Not ability-grouping, or pre-judging students.
- **Instill in students positive, growth mindset beliefs.**
- **Foster deep, conceptual understandings of mathematics.**
  - Girls tend to want to deeply understand the math concepts, so they get frustrated and anxious about procedural, superficial math.
- **Create engaging math programs for girls by having...**
  - Group work and opportunities to work together.
  - Real-life applications of math.
  - Projects and hands-on learning experiences.
- **Encourage collaboration:** Working on math collaboratively helps protect students from internalizing failure and solitary struggle.
- **Role models.**
  - Be a positive math role-model for students, especially as a female elementary teacher. Young girls look up to their teachers. Be a "math person", so that students can see themselves that way too.
  - Highlight the achievements of successful women and minorities in mathematics and STEM.
- **Eliminate homework.**
  - Homework has not been shown to improve academic achievement. It potentially even has a negative impact on achievement.
  - Homework propagates inequalities because some students, especially those of low socio-economic status, do not have the ideal environment or support to complete homework.
  - Homework takes away from family time, sports, friends, and play.

References

1 Skemp, R. R. (2006). Relational understanding and instrumental understanding. *Mathematics Teaching in The Middle School*, (2), p. 88-95.

2 Groth, R. E. (2017). Classroom data analysis with the five strands of mathematical proficiency. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas. 90*(3). 103-109.

2 Suh, J. M. (2007). Tying it all together: Classroom practices that promote mathematical proficiency for all students. *Teaching Children Mathematics. 14*(3), 163-169.

3 Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching*. San Francisco, CA: Jossey-Bass. Chapter 8.

4 Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching*. San Francisco, CA: Jossey-Bass. Chapter 7.

5 Hewitt, D. (1999). Arbitrary and necessary part 1: A way of viewing the mathematics curriculum. *For the Learning of Mathematics, 19*(3), p. 2-9.

6 Smith, M. S., & Stein, M. K. (1998). Reflections on practice: Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*(5), 344-350.

7 Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching*. San Francisco, CA: Jossey-Bass. Chapter 5.

8 Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching*. San Francisco, CA: Jossey-Bass. Chapter 2..

9 Holm J. (2018). Planning a unit by starting with the end in mind: Unit and lesson planning. In: A. Kajander, J. Holm, E. Chernoff (Eds), *Teaching and learning secondary school mathematics* (p. 441-452). Advances in Mathematics Education. Cham, Switzerland: Springer.

10 Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching*. San Francisco, CA: Jossey-Bass. Chapter 6..